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EXPERIMENTAL NOZZLE EXPANSION
AND FLOW CHARACTERISTICS
OF POTASSIUM VAPOR

by Louis J. Goldman and Stanley M. Nosek

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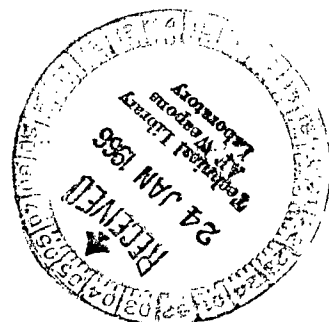Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

An experimental investigation was undertaken to determine the nozzle expansion and flow characteristics of potassium vapor. The pressure profiles through a convergent-divergent nozzle were measured using an inert-gas-injection technique. The critical weight-flow rates for potassium vapor were determined by use of a volumetric measurement technique.

The nozzle inlet pressure was varied from approximately 6 to 20 pounds per square inch absolute, which corresponded to saturation temperatures of 1240° to 1450° F. Saturated vapor and vapors with up to 250° of indicated superheat were studied. The receiving pressure was maintained at a sufficiently low value to assure supersonic flow through the nozzle.

Comparison of the experimental data for saturated inlet conditions with three theoretical expansion processes (equilibrium, supersaturated-equilibrium chemical species, and supersaturated-frozen chemical species) indicated that a supersaturated-equilibrium chemical species process best represented the data over the range investigation. An expansion index n of 1.4 (from the equation $Pv^n = c$, where P is the absolute pressure, v is the specific volume, and c is a constant) was obtained from the pressure profile data. The critical weight-flow rate data were fit by the equation $W/A_t = 2.135 \, p_o$ over the range investigated, where W is the isentropic weight-flow rate, A_t is the nozzle throat area, and p_o is the nozzle inlet pressure. For superheated inlet conditions the expansion index n was found to increase and the critical weight-flow rate to decrease with increasing superheat in accordance with theoretical considerations.

INTRODUCTION

Advanced propulsion systems for interplanetary flight will require nuclear power sources in the megawatt range. One of the methods considered for the indirect conver-

sion of nuclear energy to electric power is the Rankine conversion system utilizing an alkali metal as the working fluid. The turbine is a major component of the Rankine system and the performance of the turbine has been shown to have a significant effect on the system weight (refs. 1 and 2). In order to insure maximum turbine performance, a knowledge of the expansion and flow characteristics of the fluid is required to properly design turbine flow passages.

Since this information was not available, an experimental program was initiated at NASA Lewis Research Center. A two-phase flash-vaporization facility (described in ref. 3) was used to study the expansion and flow characteristics through a convergent-divergent nozzle for the alkali metals. The first phase of the investigation (ref. 4) concentrated on the expansion characteristics of sodium vapor. Comparison of the experimental data with a theoretical isentropic expansion indicated that an expansion index n of 1.3 (from the equation $Pv^n = c$) best represented the data. (Symbols are defined in appendix A.) The second phase of the investigation (ref. 5) was concerned with the flow characteristics of sodium vapor through a nozzle. Comparison of the experimental results with the two limiting processes (equilibrium and supersaturated-frozen) indicated that the data fell between these processes. A third theoretical process (supersaturated-equilibrium), wherein the vapor is supersaturated but the molecular species are in equilibrium, was suggested as a possible representation of the data.

Experimental results on the expansion characteristics of wet potassium vapor are reported in reference 6. The measured polytropic exponents for wet potassium vapor with up to 18 percent liquid was found to agree well with theoretical values calculated by considering a wet supersaturated flow process. In order to study the expansion characteristics of saturated as well as superheated potassium vapor the flash-vaporization facility at Lewis, used for the investigation of sodium vapor, was modified to accommodate potassium. The results of the potassium investigation are presented herein.

Pressure distributions through an axisymmetrical convergent-divergent nozzle were measured by using an inert-gas-injection technique described in reference 4. The critical weight-flow rate was determined by using a volumetric method described in reference 5.

The nozzle inlet pressure was varied from approximately 6 to 20 pounds per square inch absolute at saturation temperatures and with up to 250° of indicated superheat. The receiving pressure was maintained at a sufficiently low value to assure supersonic flow through the nozzle. The experimental results were then compared with the three theoretical processes mentioned previously.
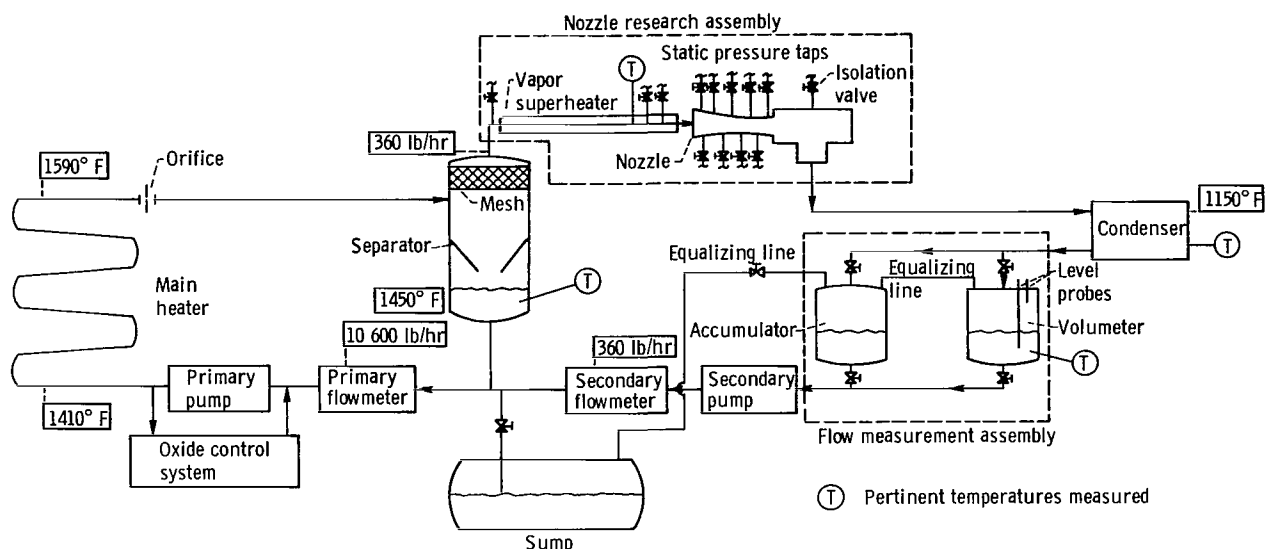
Figure 1. - Schematic drawing of potassium flash vaporization facility.

EQUIPMENT

Vapor-Generation System

The investigation was performed in a two-phase flash-vaporization facility. A brief discussion of the facility operation will be given herein with reference to figure 1, which indicates the major components and peak design parameters.

Liquid is circulated by an electromagnetic pump from a liquid-vapor separator to the main heater. The compressed liquid is heated in the electric-resistance heater. Fractional vaporization of the liquid due to flow through the orifice located at the heater outlet results in the formation of a two-phase mixture. The mixture is separated, and the liquid portion passes out the bottom and returns to the pump. The purity of the liquid is maintained by the oxide control system located across the pump.

Vapor passes from the separator, through a mesh demister, swirler, superheater, convergent-divergent nozzle, and then to an air-cooled condenser. The saturated condensate then enters either the volumeter or accumulator. The condensate then flows through the secondary pump and flowmeter before combining with the primary liquid flow from the separator. A detailed discussion of the facility and its operations (with sodium) is given in references 3, 4, and 5.

In order to carry out the research reported herein the oxide control and indicating system was modified to permit the use of potassium as the working fluid. The cold trap and plugging indicator were removed and the hot trap (14-in. length of 3-in.-diam pipe) was replaced by a larger one consisting of an 18-inch length of 12-inch-diameter pipe.
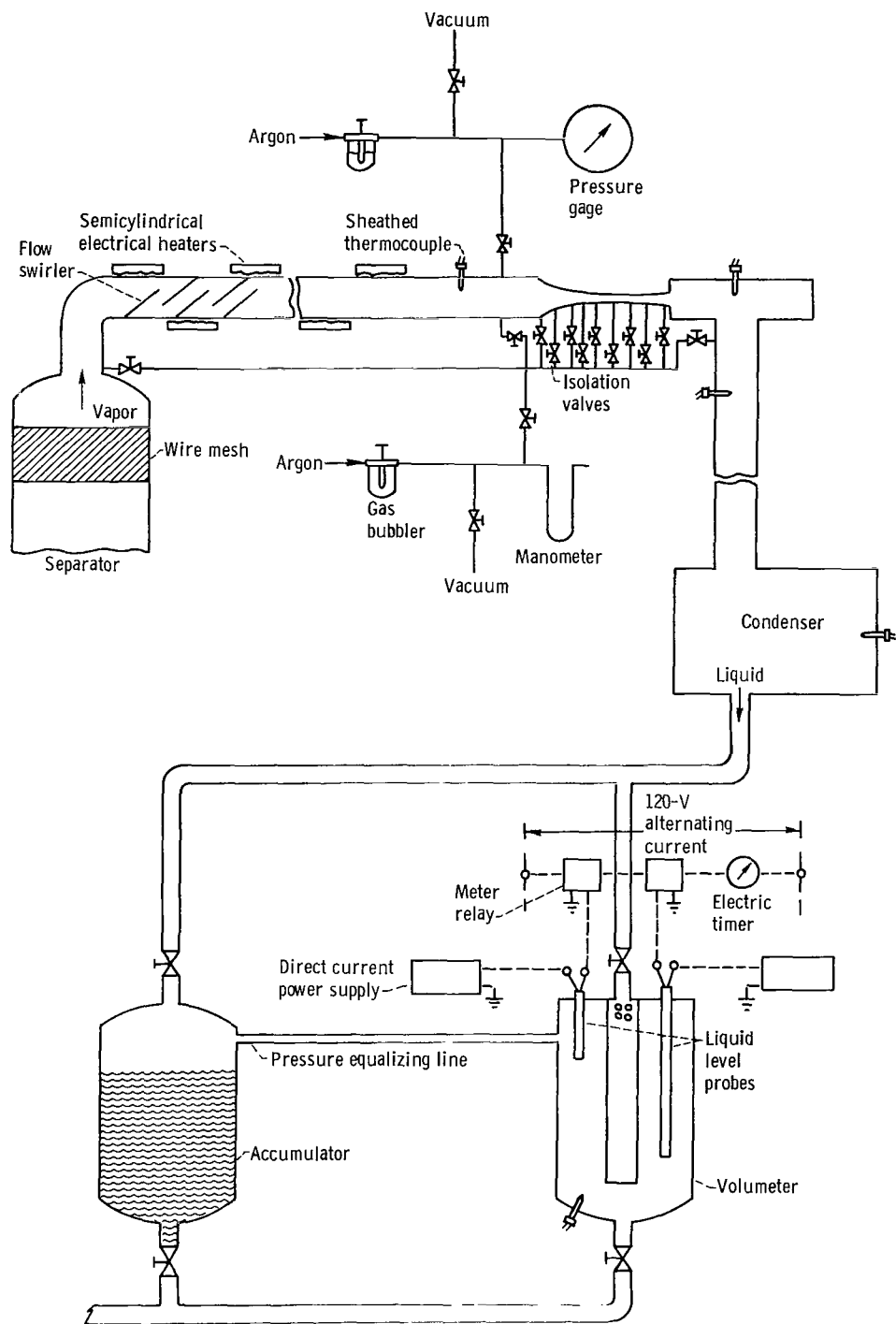
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Figure 2. - Schematic drawing of research assembly.

The 0.7-inch-diameter sharp-edge orifice was replaced by a similar one but with a 0.6-inch diameter to insure sufficient pressure upstream to prevent boiling in the heater.

Research Assembly

The research assembly, shown schematically in figure 2, is composed essentially of an axisymmetrical convergent-divergent nozzle with wall static-pressure taps, an inert-gas-injection pressure measuring assembly, and a volumetric flow measuring device.

Nozzle. - A schematic diagram of the nozzle is shown in figure 3. The convergent section of the nozzle is elliptical in contour and fairs slightly downstream of the throat into a conical divergent section. The nozzle static-pressure taps are 0.020-inch burr-free, square-edge holes. The inlet and outlet pressure taps are 1/16-inch holes. A complete description of the nozzle and its fabrication is given in reference 4.

Pressure-measuring technique. - An inert-gas-injection technique was used to measure the nozzle static-pressures. By this method, the pressure is null-balanced with an inert gas barrier so that readings can be taken with a mercury manometer. To effect a pressure reading the inert-gas pressure is set slightly higher than the predicted system pressure, and the appropriate isolation valve is opened. Any condensed potassium present between the isolation valve and the 0.020-inch wall tap is forced out into the flowing potassium vapor stream. Pressures are recorded when no further change is noted in the mercury column. The pressure measuring technique is described fully in reference 4.

For this investigation two additional pressure taps were added, one at the separator outlet and the other at the nozzle inlet. One of the duplicate taps at the nozzle inlet was equipped with its own pressure gage, which enabled continuous visual observation of the inlet pressure during the course of taking a complete set of nozzle static-pressure data.

Vapor flow measurement. - The vapor flow rate was determined by a volumetric technique. The vapor was first condensed and allowed to drain freely into a volumeter. Then the time of rise between electrical-resistance, on-off type, liquid-level probes was measured. The liquid in the accumulator meanwhile was used to maintain the primary system in steady state. A description of the vapor-flow measurement system and its operation is given in reference 5.
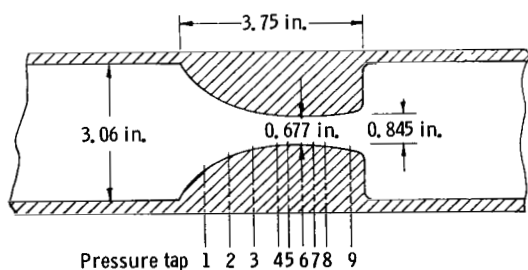
Pressure tap	Distance from inlet, in.	Nozzle diameter, in.
1	0.52	1.584
2	1.02	1.129
3	1.52	.866
4	2.02	.721
5	2.27	.689
6 (Throat)	2.52	.677
7	2.77	.691
8	3.02	.725
9	3.52	.803

Figure 3. - Diagrammatic sketch and dimensions of nozzle.

DATA ANALYSIS TECHNIQUE

Reduction of Experimental Data

Nozzle static-pressure data. - The static-pressure data for potassium vapor in the range from approximately 6 to 18 pounds per square inch are tabulated in table I. For each set of data the inlet pressure was visually noted not to vary more than 0.1 pound per square inch during the course of the run. All static wall tap pressure readings except separator, nozzle inlet, and condenser pressures are listed as pressure ratios P_x/P_o. The measured inlet pressure p_o was used to obtain the pressure ratios in all cases except runs 10 and 22, where the inlet tap was plugged. The value of the inlet pressure for these two cases was obtained by extrapolation of the available pressure data to the nozzle inlet. Also listed in table I are the pertinent temperature data taken during each set of pressure readings. The amount of superheat, if superheat was applied, is listed with the temperature data. The expansion indexes n and n_t (calculated by assuming the expansion follows the relation $Pv^n = c$) are also listed in table I. The expansion index n was obtained by graphically minimizing the sum of the squares of the differences between the observed values of P_x/P_o and the theoretical pressure ratios calculated from the following equation (ref. 4) for various values of n:

$$\frac{A_x}{A_t} = \frac{\left[\left(\frac{n-1}{n+1}\right)\left(\frac{2}{n+1}\right)^{2/(n-1)}\right]^{1/2}}{\left[\left(\frac{P_x}{P_o}\right)^{2/n} - \left(\frac{P_x}{P_o}\right)^{(n+1)/n}\right]^{1/2}} \tag{1}$$

The value of the expansion index at the throat n_t was calculated from the critical pressure ratio by using the equation

$$\frac{P_t}{P_o} = \left(\frac{2}{n_t + 1}\right)^{n_t/(n_t-1)} \tag{2}$$

Critical weight-flow data. - The critical weight-flow data for potassium vapor over the range of nozzle inlet pressures of approximately 6 to 20 pounds per square inch are presented in table II. The critical weight-flow rate was calculated from the equation

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$$\frac{W}{A_t} = \frac{w}{C_D A_t} = \frac{Q\rho}{C_D A_t \theta} \tag{3}$$

The liquid density ρ as a function of temperature was obtained from reference 7. The volume Q of the volumeter and the throat area A_t were corrected for temperature growth by using the thermal expansion data (for AISI 316 stainless steel) of reference 8. The volume and throat area at 70° F are 0.4523 cubic foot and 0.002411 square foot, respectively. The nozzle discharge coefficient C_D was determined by pretesting with air, as described in reference 4. For the Reynolds number range of this investigation a constant value of 0.97 was considered reasonable for all nozzle inlet conditions. Since a large number of duplicate runs was taken, the arithmetic average value of the critical weight-flow rate as well as the average deviation are tabulated in table II. Pertinent temperature data taken during each set of weight-flow determinations are presented in table II as is the amount of superheat, if superheat was applied.

The nozzle inlet pressure p_0 is also listed in table II. The nozzle inlet pressure was not measured for every inlet temperature condition for which the critical weight-flow was measured. To utilize all the critical weight-flow data the experimental nozzle inlet pressure and inlet temperature (or separator temperature) data of table I were used to obtain the equation

$$\log p_0 = \frac{-7855.0}{T} + 5.40461 \tag{4}$$

The nozzle inlet pressure for all the critical weight-flow runs was then calculated from equation (4). The experimental inlet pressure and temperature data are compared in figure 4 with a plot of equation (4) and the vapor-pressure curve of reference 9. The experimental points and the plot of equation (4) differ by approximately 0.5 percent in temperature from that of the vapor-pressure curve of reference 9. This is within the accuracy of the thermocouples used.

Figure 4. - Comparison of measured pressures with vapor pressure equation for potassium.

Comparison with Theoretical Processes

In order to determine a means for pre-

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dicting the nozzle expansion and flow characteristics of potassium vapor the experimental results have been compared with theoretical expansion processes. Three possible expansion processes were considered: (1) equilibrium, (2) supersaturated-equilibrium chemical species, and (3) supersaturated-frozen chemical species. The calculation of the pressure profiles and resulting expansion indexes n, as well as the critical weight-flows, as functions of nozzle inlet condition were made for all processes assuming one-dimensional, isentropic flow. The calculation procedure for the three processes is given in appendix B. The results indicated that the three theoretical processes can be represented by expansions following the equation $Pv^n = c$. The expansion indexes n and the critical weight-flow rates that approximate the processes are summarized in the following table:

| Expansion process	Expansion index, n (a)	Critical weight-flow rate, W/A_t (a)
Equilibrium	1.2	$(W/A_t)_E = 2.029\ p_0$
Supersaturated-equilibrium chemical species	1.4	$(W/A_t)_{SE} = 2.158\ p_0$
Supersaturated-frozen chemical species	1.6	$(W/A_t)_{SF} = 2.265\ p_0$

[a]Approximate representation.

RESULTS AND DISCUSSION

Saturated Nozzle Inlet Conditions

Expansion characteristics. - The variation of pressure through the nozzle for saturated inlet conditions is shown in figure 5(a) for four runs made with the nozzle inlet pressures ranging from 6.43 to 17.88 pounds per square inch. The location of the nozzle taps, presented in figure 3 (p. 5) have been corrected for the thermal growth of the nozzle. Values of the expansion index n of 1.2, 1.4, and 1.6 were used to represent the three theoretical processes. For clarity, only the highest inlet pressure run shows all three values of n. The experimental data can be best represented by the theoretical supersaturated-equilibrium chemical species process (n = 1.4) except for the last two taps. This departure in the supersonic portion of the nozzle may be due to the formation of moisture and the consequential change to an equilibrium flow process.

All the pressure data for saturated inlet conditions are shown as the variation of pressure ratio through the nozzle in figure 5(b). At each wall tap, the average pressure ratio is shown. The three theoretical processes are represented by dashed lines. As in
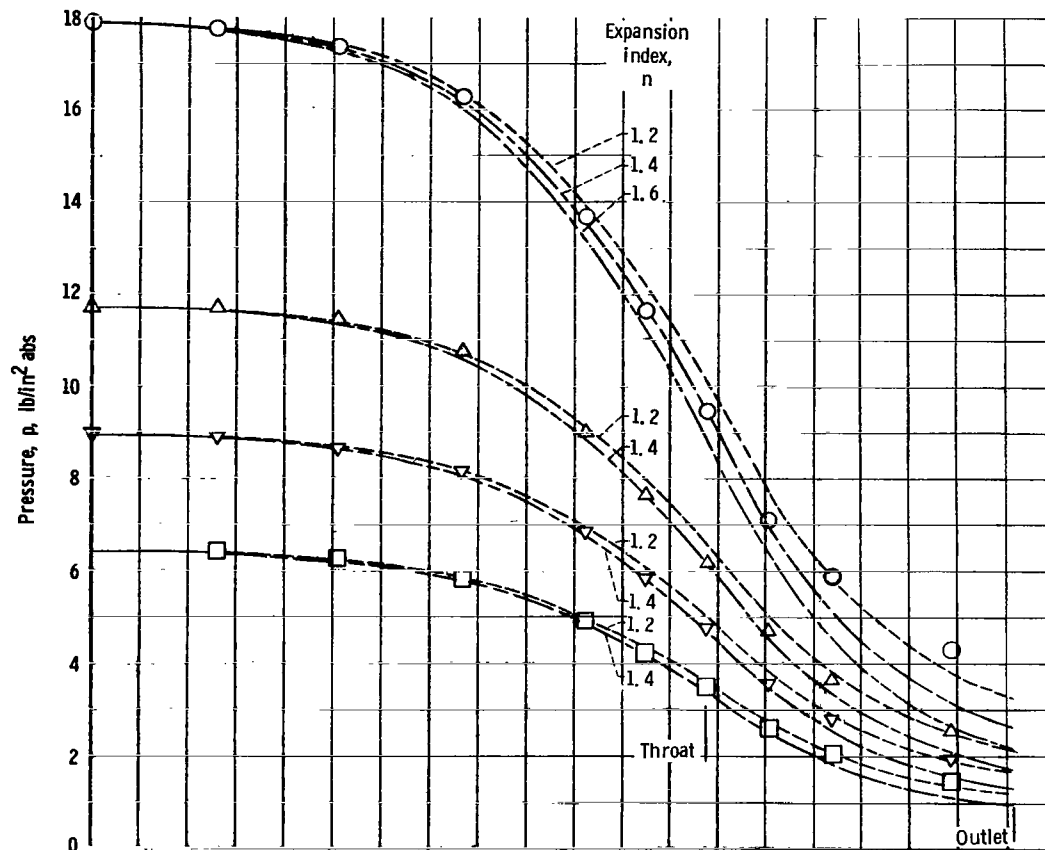
(a) Variation of static-pressure measurements through nozzle.
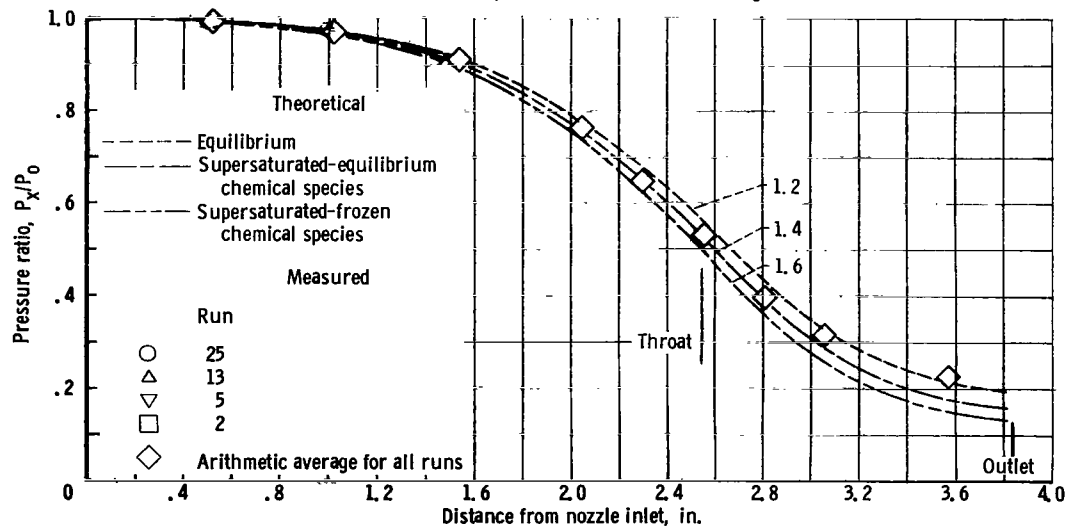
(b) Variation of static- to inlet-pressure ratio along nozzle wall.

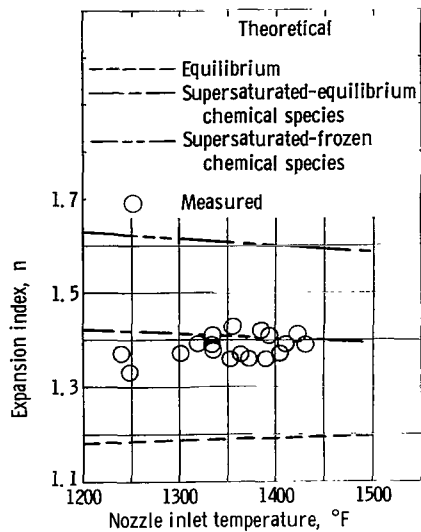Figure 5. – Pressure data for saturated potassium vapor.

Figure 6. - Variation of expansion index with nozzle inlet temperature for saturated potassium vapor.

figure 5(a), the experimental data falls close to a supersaturated-equilibrium chemical species process (n = 1.4) except for the last two taps.

The variation of the expansion index n with nozzle inlet temperature for saturated inlet conditions is shown in figure 6. The three theoretical processes are represented by dashed lines. The experimental expansion index was calculated from the measured pressure ratios, as described in the section Reduction of Data. The expansion index at the throat n_t, which was essentially the same value as n, is tabulated in table I but is not shown in figure 6. Except for the low value of the expansion index at 1250° F the experimental points can be best represented by a theoretical supersaturated-equilibrium chemical species process. Since no significant trend in expansion index is noted, all the data have been consolidated to determine average values of n and n_t. From the average pressure ratios at each tap (for all runs taken at saturated inlet conditions) an expansion index n of 1.38 has been determined. Similarly, the average pressure ratio at the throat resulted in a value of n_t of 1.38. It should be noted that the expansion index, as determined herein, includes the effect of friction in the nozzle; for example, with an estimated nozzle efficiency of 96 percent, the isentropic index would be about 1.41.

The experimental expansion characteristics of wet potassium vapor over the temperature range from 1450° to 1580° F are reported in reference 6. For runs made at high inlet quality (98 percent) expansion indexes between 1.42 and 1.46 were determined. The expansion index was found to decrease as the inlet quality decreased. Also of interest was the observed increase in pressure ratio downstream of the nozzle throat, which was attributed to the reversion of the expansion process to an equilibrium one. The results of this investigation are essentially consistent with those of reference 6.

Flow characteristics. - The variation of the critical weight-flow rate of potassium vapor with nozzle inlet pressure for saturated inlet conditions is shown in figure 7. The three theoretical processes are represented by dashed lines. For a comparison of the measured flows with the theoretical flows it is assumed that the vapors entering the nozzle are saturated. No direct measurement of the vapor quality was made, but a heat balance calculation indicated that the quality was greater than 98 percent. For duplicate runs the average value of the experimental weight flow are plotted. All the weight-flow data however were used to obtain the least squares equation:

$$\frac{W}{A_t} = 2.135\, p_o \qquad (5)$$

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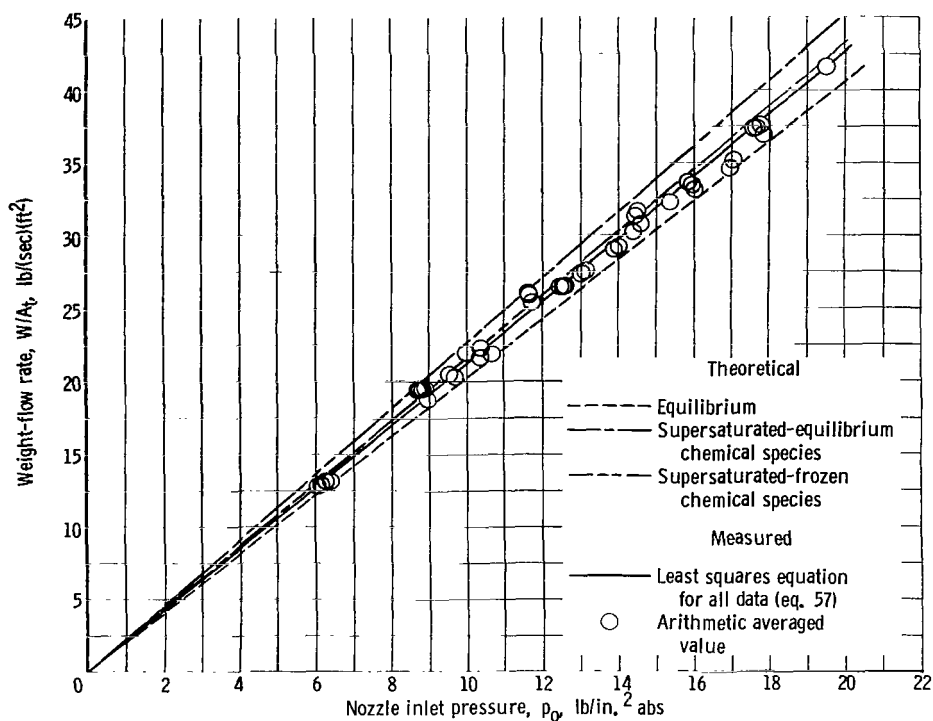
Figure 7. - Variation of critical weight-flow rate with nozzle inlet pressure for saturated potassium vapor.

which is represented by the solid line in figure 7. Comparison of the experimental data, as expressed by the least squares equation, with the three theoretical processes indicates that the data is best represented by the supersaturated-equilibrium chemical species process over the range investigated.

Superheated Nozzle Inlet Conditions

Expansion characteristics. - The variation of the pressure through the nozzle for superheated inlet conditions is shown in figure 8 for three runs made with the inlet pressure ranging from 6.60 to 12.17 pounds per square inch. The amount of superheat ranged from 194° to 249°. A value of the expansion index of 1.5 was used to represent the theoretical supersaturated-equilibrium chemical species process at these superheat conditions. As for saturated inlet conditions, the experimental data can be represented by the supersaturated-equilibrium chemical species process except for the last two taps.

The variation of the expansion index n with the degrees of superheat for nozzle inlet pressures ranging from 6.47 to 14.64 pounds per square inch are shown in figure 9. The theoretical supersaturated-equilibrium chemical process for the limits of the experimental pressure range are represented by dashed lines in figure 9. The theoretical

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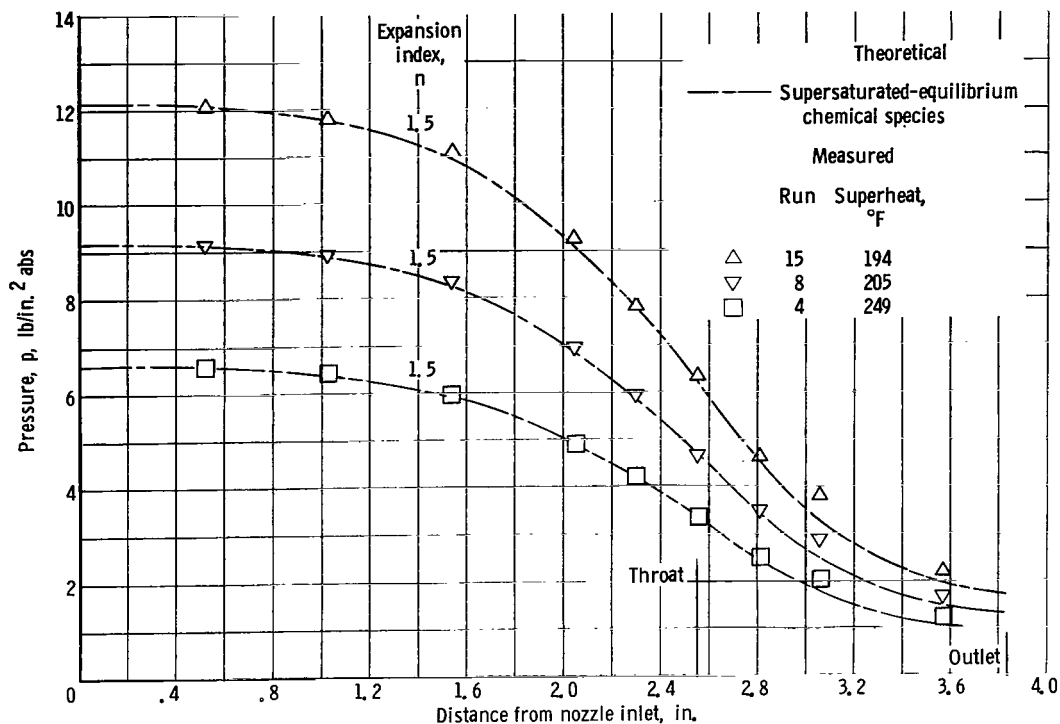
Figure 8. - Variation of static-pressure measurements through nozzle for superheated potassium vapor.
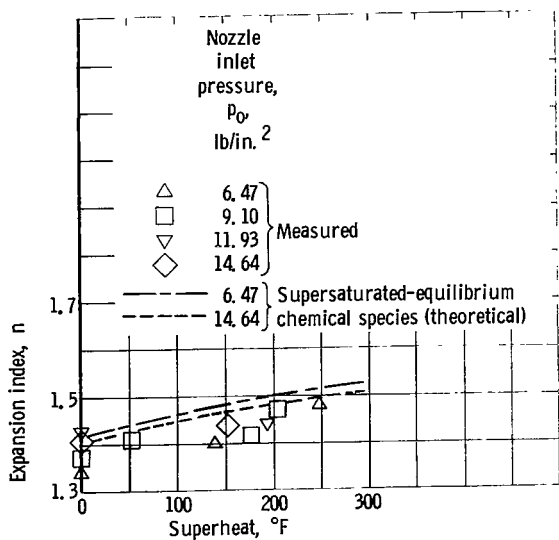
Figure 9. - Variation of expansion index with superheat for potassium vapor.

expansion index increases approximately 2.5 to 3.0 percent per 100° of superheat. Although the experimentally determined expansion indexes fall below the theoretical curves, the data do show an increasing trend with superheat.

Flow characteristics. - The variation of the critical weight-flow rate of potassium vapor with nozzle inlet temperature for saturated and superheated inlet conditions is shown in figure 10. The experimental data at superheated inlet conditions varied over the pressure range from 6.47 to 14.64 pounds per square inch. The theoretical supersaturated-equilibrium chemical species process is represented by dashed lines. For superheated inlet conditions the theoretical weight-flow decreases
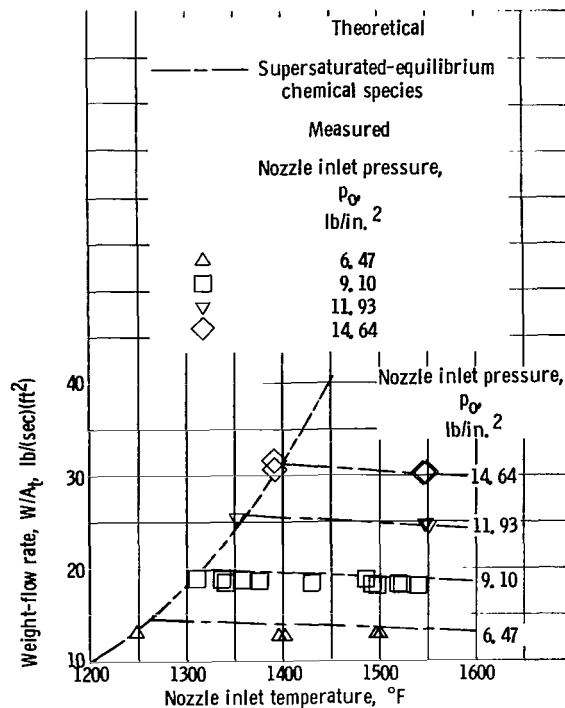
Figure 10. - Variation of critical weight-flow rate with nozzle inlet temperature for saturated and superheated potassium vapor.

approximately 2. 5 to 3. 0 percent per 100° of superheat. For the inlet condition investigated most extensively (i. e. , inlet pressure of 9. 10 psi) the experimental data indicate a decreasing trend in the weight-flow rate with superheat, although the data does fall below the theoretical curve. The other three superheated conditions were not completely investigated experimentally.

SUMMARY OF RESULTS

The critical weight-flow rates and nozzle pressure profiles for potassium vapor flowing through a nozzle were experimentally determined over a nozzle inlet pressure range of approximately 6 to 20 pounds per square inch absolute. Saturated vapor and vapors with up to 250° of indicated superheat were studied. The experimental data were compared with different theoretical processes (equilibrium, supersaturated-equilibrium chemical species, and supersaturated-frozen chemical species). The following conclusions were reached:

1. The theoretical expansion processes (equilibrium, supersaturated-equilibrium chemical species, and supersaturated-frozen chemical species) considered herein for

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potassium vapor can be represented by an expansion following the equation $Pv^n = c$, where P is the absolute pressure, v is the specific volume, n is the expansion index, and c is a constant. The approximate values of the expansion index n for potassium are 1.2, 1.4, and 1.6, respectively.

2. For saturated inlet conditions the experimental nozzle expansion and flow characteristics of potassium vapor were found to be best represented by a theoretical supersaturated-equilibrium chemical species process. A value of 1.4 for the expansion index n was obtained from pressure profile data. The critical weight-flow data were fit by the equation $W/A_t = 2.135 \, p_o$ over the range investigated, where W is the isentropic weight-flow rate, A_t is the nozzle throat area, and p_o is the nozzle inlet pressure.

3. For superheated inlet conditions the expansion index was found to increase and the critical weight-flow rate to decrease with increasing superheat in accordance with theoretical considerations.

Lewis Research Center,
    National Aeronautics and Space Administration,
        Cleveland, Ohio, September 12, 1965.

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APPENDIX A

SYMBOLS

A area, ft^2

B second virial coefficient, ft^3/lb

C third virial coefficient, ft^5/lb^2

C_D nozzle discharge coefficient, ratio of actual to isentropic flow rate

C_p molal specific heat at constant pressure, $Btu/(lb\text{-}mole)(^\circ R)$

C_v molal specific heat at constant volume, $Btu/(lb\text{-}mole)(^\circ R)$

c constant

g acceleration of gravity, 32.17 ft/sec^2

J mechanical equivalent of heat, 778.2 ft-lb/Btu

M molecular weight of monomer

n expansion index

P absolute pressure, lb/ft^2

p absolute pressure, $lb/in.^2$

Q volume, ft^3

R universal gas constant, 1545 $ft\text{-}lb/(lb\text{-}mole)(^\circ R)$

T temperature, $^\circ R$

V velocity, ft/sec

v specific volume, ft^3/lb

W isentropic weight-flow rate, lb/sec

w actual weight-flow rate, lb/sec

X mole fraction

θ time of flow, sec

ρ density, lb/ft^3

Subscripts:

E equilibrium

f frozen

j species j

o inlet of nozzle

P pressure

SE supersaturated-equilibrium chemical species

SF supersaturated-frozen chemical species

s entropy

T temperature

t throat of nozzle

x variable state point

1 monomer species

APPENDIX B

THEORETICAL CALCULATION OF NOZZLE EXPANSION AND
FLOW CHARACTERISTICS OF POTASSIUM VAPOR

The calculation of the nozzle expansion and flow characteristics of potassium vapor have been made by assuming one-dimensional, isentropic flow. The pressure profiles and resulting expansion indexes n as well as the critical weight-flows were determined for different expansion processes. The three processes considered are (1) equilibrium, (2) supersaturated-frozen chemical species, and (3) supersaturated-equilibrium chemical species.

Equilibrium Expansion Process

During the equilibrium expansion of a vapor below the saturated state, the expansion is assumed to occur with equilibrium both between the physical phases of vapor and liquid resulting from condensation and between the chemical species of monomer, dimer, etc., in the vapor phase resulting from the polymerization reaction.

The weight-flow rate at various inlet pressures was determined from the continuity equation:

$$\left(\frac{W}{A_x}\right)_E = \left(\frac{V_x}{v_x}\right)_E \tag{B1}$$

For different values of pressure ratio P_x/P_o, the value for $(V_x/v_x)_E$ was calculated and used to determine the maximum or critical weight-flow rate $(W/A_t)_E$. The equilibrium expansion calculation procedure is described fully in reference 3. The calculations for potassium were made by using the experimental thermodynamic data of reference 9. The results for an equilibrium process showed that the critical weight-flow rate as a function of nozzle inlet pressure could be represented, over the pressure range used in this report, by the equation

$$\left(\frac{W}{A_t}\right)_E = 2.029\ p_o \tag{B2}$$

The pressure variation during the expansion can be obtained by calculating the area ratio A_x/A_t for various pressure ratios from the equation

$$\frac{A_x}{A_t} = \frac{\left(W/A_t\right)_E}{\left(W/A_x\right)_E} \tag{B3}$$

For the equilibrium process it was found that the variation of pressure ratio with area ratio could be represented by assuming that the expansion follows the equation $Pv^n = c$. The variation of pressure ratio could therefore be calculated more conveniently by utilizing the equation (ref. 4):

$$\frac{A_x}{A_t} = \frac{\left[\left(\frac{n-1}{n+1}\right)\left(\frac{2}{n+1}\right)^{2/(n-1)}\right]^{1/2}}{\left[\left(\frac{P_x}{P_o}\right)^{2/n} - \left(\frac{P_x}{P_o}\right)^{(n+1)/n}\right]^{1/2}} \tag{B4}$$

The expansion index for the equilibrium process n_E was found to be approximately 1.4.

Supersaturated-Frozen Chemical Species Expansion Process

During the supersaturated-frozen chemical species expansion neither condensation nor reaction among the chemical species in the vapor phase was assumed to occur. This results in a vaporous expansion in which the molecular weight of the vapor remains unchanged from the initial-state point.

If molecular weight changes due to the polymerization reaction are assumed to be the only factor contributing to the nonideality of potassium vapor, then for a supersaturated-frozen chemical species expansion (molecular weight constant) the vapor can be expected to behave as an ideal or perfect gas. For an isentropic expansion of a perfect gas the pressure and specific volume are related by the equation

$$Pv^{C_{p,f}/C_{v,f}} = c \tag{B5}$$

where

$$C_{v,f} = C_{p,f} - \frac{R}{J}$$

The expansion index n_{SF} is equal to $C_{p,f}/C_{v,f}$. The calculation of n_{SF} requires a knowledge of the frozen specific heat $C_{p,f}$ of the vapor. For a mixture, such as potassium, $C_{p,f}$ can be calculated from

$$C_{p,f} = \sum_j x_j C_{p,j} \tag{B6}$$

where the $C_{p,j}$'s are the specific heats of the ideal polyatomic species. The existence of diatomic and triatomic species in the vapor phase has been reported in reference 10. The specific heats of ideal monatomic and diatomic potassium vapor are available in reference 11, and values for the triatomic species were extrapolated from these. The frozen specific heat was calculated from equation (B6) by using the vapor composition obtained from reference 10. The value of n_{SF} resulting from the calculation of $C_{p,f}/C_{v,f}$ is approximately 1.6.

The critical weight-flow rate for a perfect gas expanding isentropically can be expressed as

$$\frac{W}{A_t} = \left[\left(\frac{2gn}{n+1} \right) \left(\frac{P_o}{v_o} \right) \left(\frac{2}{n+1} \right)^{2/(n-1)} \right]^{1/2} \tag{B7}$$

The critical weight-flow rate $(W/A_t)_{SF}$ for a supersaturated-frozen chemical species process can be calculated from equation (B7) by using n_{SF}. Results indicated that over the pressure range used in this report the critical weight-flow rate could be represented by the equation

$$\left(\frac{W}{A_t} \right)_{SF} = 2.265 \, p_o \tag{B8}$$

which is 11.6 percent greater than flow for an equilibrium process.

The variation of the pressure ratio with area ratio for this process can be calculated from equation (B4) by using n_{SF}.

Supersaturated-Equilibrium Chemical Species Expansion Process

During the supersaturated-equilibrium chemical species expansion the fluid remains

vaporous and equilibrium among the chemical species is assumed to occur.

The weight-flow rate for a gas expanding isentropically can be represented in the general form as (ref. 4)

$$\frac{W}{A_x} = \frac{1}{v_x}\left[2g\left(P_0 v_0 - P_x v_x + \int_{v_0}^{v_x} P \, dv\right)\right]^{1/2} \tag{B9}$$

The calculation of the critical weight-flow rate $(W/A_t)_{SE}$ for a supersaturated-equilibrium chemical species process from equation (B9) requires the variation of the pressure and specific volume during the expansion so that the term $\int_{v_0}^{v_x} P \, dv$ can be determined. This can be accomplished through a knowledge of the equation of state for potassium. An experimental equation of state reported in reference 9 was used for this purpose. In slightly different form it can be represented as

$$Pv = \frac{RT}{M} + BP + CP^2 \tag{B10}$$

where B and C are functions of temperature only. The variation of pressure and specific volume during the expansion was calculated by first evaluating the variation of pressure and temperature and then calculating the specific volume from equation (B10). The pressure and temperature variation for an isentropic expansion of a gas can be represented (ref. 12) by the equation

$$\left(\frac{\partial P}{\partial T}\right)_s = \frac{J}{M}\left(\frac{C_p}{T}\right)\left(\frac{\partial T}{\partial v}\right)_P \tag{B11}$$

The partial derivative $(\partial T/\partial v)_P$ is obtained from the equation of state (eq. (B10)). The specific heat C_p as a function of temperature and pressure can be obtained from the equation (ref. 12)

$$\left(\frac{\partial C_p}{\partial P}\right)_T = -\frac{MT}{J}\left(\frac{\partial^2 v}{\partial T^2}\right)_P \tag{B12}$$

by determining $(\partial^2 v/\partial T^2)_P$ from equation (B10) and integrating equation (B12) at con-

stant temperature between the pressure limits of zero and P. Substituting into equation (B11) results in

$$\left(\frac{\partial P}{\partial T}\right)_s = \frac{\dfrac{JC_{p,1}}{MT} - P\dfrac{d^2B}{dT^2} - \dfrac{P^2}{2}\dfrac{d^2C}{dT^2}}{\dfrac{R}{MP} + \dfrac{dB}{dT} + P\dfrac{dC}{dT}} \tag{B13}$$

where $C_{p,1}$ is the specific heat of ideal monatomic vapor at zero pressure. Equation (B13) can at constant entropy be numerically integrated and the specific volume computed from equation (B10). The weight flow can then be calculated from equation (B9) by numerically integrating the term $\int_{v_0}^{v_x} P\,dv$. An iterative procedure is then used to find the maximum or critical weight-flow rate $(W/A_t)_{SE}$. The variation of pressure ratio and area ratio during the expansion can be obtained from equation (B3). The results of these calculations indicated that the supersaturated-equilibrium chemical species expansion could be represented by a process following the equation $Pv^n = c$. The expansion index n_{SE} was found to be approximately 1.4.

Since the expansion follows the equation $Pv^n = c$, the expansion index n_{SE} can be calculated more conveniently by differentiating at constant entropy the equation $Pv^n = c$. This differentiation results in

$$n = -\frac{v}{P}\left(\frac{\partial P}{\partial v}\right)_s \tag{B14}$$

which can also be represented as (ref. 12)

$$n = -\frac{\dfrac{v}{P}}{\left(\dfrac{\partial v}{\partial P}\right)_T + \dfrac{MT}{JC_p}\left(\dfrac{\partial v}{\partial T}\right)_P^2} \tag{B15}$$

The partial derivatives $(\partial v/\partial P)_T$ and $(\partial v/\partial T)_P$ are obtained from equation (B10). Values of n can be calculated for both saturated and superheated potassium vapor. The critical weight-flow rate can be calculated from equation (B7) by using n_{SE}. The critical weight-flow rate over the pressure range used in this report can be represented by the equation

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$$\left(\frac{W}{A_t}\right)_{SE} = 2.158 \, p_o \qquad\qquad (B16)$$

This value for the critical weight-flow rate is 6.4 percent greater than that for an equilibrium process. The variation of pressure ratio with area ratio can also be calculated more conveniently from equation (B4) by utilizing the value of n_{SE}.

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TABLE I. - NOZZLE PRESSURE DATA

Run	Temperature, ^{O}F			Pressure, psia			Pressure ratio, P_x/P_o											Process expansion index, η	Throat expansion index, η_t
	Sepa-rator	Super-heat	Conden-ser	Sepa-rator	Nozzle inlet	Conden-ser	Nozzle wall tap									Conden-ser		
							1	2	3	4	5	6	7	8	9			
1	1238	0	1046	6.00	6.02	1.64	0.990	0.967	0.903	0.758	0.644	0.545	0.403	0.308	0.241	0.273	1.37	1.31
2	1248	0	1050	6.46	6.43	1.49	1.000	.974	.905	.766	.657	.544	.408	.322	.227	.231	1.33	1.31
3	1250	139	1045	6.57	6.57	1.65	.987	.962	.900	.768	.640	.533	.395	.318	.221	.251	1.40	1.37
4	1251	249	1045	6.60	6.60	1.68	.994	.974	.906	.752	.640	.509	.380	.310	.189	.254	1.48	1.52
5	1302	0	1051	8.95	8.94	1.81	.995	.969	.911	.766	.650	.532	.402	.311	.220	.203	1.37	1.38
6	1304	53	1050	9.01	8.99	----	.998	.967	.905	.763	.651	.526	.393	.316	.204	-----	1.41	1.42
7	1306	178	1047	9.20	9.20	1.57	.985	.959	.910	.761	.644	.521	.398	.308	.183	.171	1.42	1.44
8	1307	205	1046	9.23	9.19	1.63	.994	.970	.909	.759	.643	.508	.380	.315	.182	.177	1.47	1.53
9	1316	0	1051	9.49	9.51	1.57	.991	.970	.905	.765	.649	.529	.395	.307	.211	.165	1.39	1.40
10	1331	0	1053	10.44	^{a}10.39	1.83	.994	.968	.907	.767	.649	.528	.398	.309	.216	.176	1.39	1.40
11	1333	0	1050	10.56	10.52	1.76	.994	.971	.910	.764	.648	.529	.400	.308	.219	.168	1.38	1.40
12	1335	0	1050	10.51	10.58	1.76	.992	.971	.906	.762	.648	.527	.394	.306	.213	.167	1.41	1.41
13	1353	0	1053	11.83	11.71	1.86	1.000	.981	.921	.770	.655	.529	.402	.314	.218	.158	1.36	1.40
14	1356	0	1040	12.19	12.12	----	.999	.974	.909	.760	.640	.520	.394	.326	.182	-----	1.43	1.45
15	1356	194	1042	12.19	12.17	1.64	.992	.971	.913	.762	.650	.521	.381	.313	.181	.134	1.44	1.45
16	1362	0	1050	12.41	12.33	1.90	.989	.970	.917	.767	.649	.529	.400	.310	.224	.154	1.37	1.40
17	1371	0	1052	13.17	13.10	1.87	.994	.972	.912	.769	.652	.532	.402	.331	.224	.143	1.36	1.38
18	1383	0	1051	13.79	13.76	1.95	.997	.973	.911	.762	.647	.521	.390	.306	.223	.142	1.42	1.45
19	1390	0	1050	14.73	14.63	1.99	.989	.971	.912	.766	.652	.530	.405	.330	.226	.136	1.36	1.39
20	1392	0	1053	14.55	14.47	----	.999	.977	.915	.766	.651	.522	.390	.317	.225	-----	1.41	1.44
21	1393	153	1085	15.03	15.00	2.44	.994	.969	.907	.759	.641	.514	.395	.328	.178	.163	1.44	1.49
22	1402	0	1051	-----	^{a}15.21	1.94	.995	.976	.916	.769	.652	.534	.400	.322	.237	.128	1.37	1.37
23	1409	0	1098	16.10	16.02	2.60	.998	.972	.912	.765	.649	.528	.398	.328	.237	.163	1.39	1.40
24	1422	0	1053	16.89	16.83	----	.992	.968	.909	.764	.646	.526	.393	.320	.239	-----	1.41	1.42
25	1431	0	1098	17.92	17.88	2.71	.993	.971	.911	.765	.650	.528	.397	.330	.240	.151	1.39	1.40

^{a}Extrapolated value.

TABLE II. – NOZZLE VAPOR FLOW RATE DATA

Run	Number of runs	Temperature, °F Separator	Super-heat	Conden-ser	Average weight-flow rate, W/A_t, lb/(sec)(ft^2)	Average devia-tion, $\frac{lb}{(sec)(ft^2)}$	Nozzle inlet pressure, p_o, psia (a)	Run	Number of runs	Temperature, °F Separator	Super-heat	Conden-ser	Average weight-flow rate, W/A_t, lb/(sec)(ft^2)	Average devia-tion, $\frac{lb}{(sec)(ft^2)}$	Nozzle inlet pressure, p_o, psia (a)
1	1	1239	0	1048	12.88	-----	6.04	34	4	1323	0	1053	21.94	±0.06	9.98
2	1	1240	0	1049	12.94	-----	6.08	35	1	1329	0	1053	22.27	----	10.33
3	1	1245	0	1051	13.22	-----	6.27	36	3	1330	0	1053	21.76	±.11	10.38
4	1	1246	0	1053	13.24	-----	6.31	37	5	1335	0	1051	21.91	±.26	10.68
5	3	1247	0	1052	13.20	±0.13	6.35	38	4	1350	0	1049	26.00	±.25	11.61
6	1	1248	0	1052	13.25	-----	6.39	39	4	1351	0	1053	25.81	±.16	11.67
7	3	1249	153	1049	12.74	±.24	6.43	40	3	1352	0	1053	25.47	±.38	11.74
8	3	1250	147	1047	12.78	±.07	6.47	41	1	1355	197	1038	24.40	----	11.93
9	3	1250	251	1047	12.93	±.06	6.47	42	5	1356	195	1045	24.32	±.10	12.00
10	4	1251	248	1047	12.99	±.07	6.51	43	4	1362	0	1051	26.66	±.26	12.40
11	2	1300	0	1054	19.35	±.08	8.74	44	3	1363	0	1051	26.71	±.23	12.47
12	3	1301	0	1053	19.46	±.10	8.79	45	1	1364	0	1052	26.52	----	12.54
13	2	1301	56	1049	18.93	±.05	8.79	46	3	1371	0	1054	27.45	±.14	13.02
14	6	1302	0	1051	19.23	±.35	8.84	47	4	1372	0	1055	27.50	±.10	13.09
15	1	1302	6	1049	18.72	-----	8.84	48	2	1383	0	1054	29.04	±.15	13.89
16	4	1303	0	1052	19.57	±.40	8.90	49	3	1384	0	1053	29.23	±.10	13.96
17	2	1303	7	1048	18.86	±.02	8.90	50	3	1389	0	1054	30.27	±.04	14.33
18	1	1304	0	1051	18.64	-----	8.95	51	4	1390	0	1052	31.47	±.49	14.41
19	2	1304	6	1049	18.91	±.10	8.95	52	7	1391	0	1053	31.72	±.55	14.49
20	5	1304	34	1046	18.71	±.09	8.95	53	2	1392	0	1058	30.88	±.04	14.56
21	2	1304	53	1054	18.71	±.06	8.95	54	3	1393	152	1089	30.28	±.14	14.64
22	1	1305	34	1045	18.67	-----	9.00	55	4	1394	152	1088	30.19	±.05	14.72
23	1	1305	51	1053	18.65	-----	9.00	56	5	1402	0	1052	32.29	±.09	15.35
24	1	1305	215	1047	18.36	-----	9.00	57	2	1408	0	1100	33.63	±.01	15.83
25	2	1306	181	1051	18.71	±.11	9.05	58	2	1409	0	1100	33.46	±.07	15.92
26	4	1307	68	1047	18.59	±.03	9.10	59	2	1410	0	1101	33.15	±.01	16.00
27	2	1307	122	1044	18.42	±.02	9.10	60	1	1421	0	1055	34.68	----	16.93
28	3	1307	186	1043	18.22	±.03	9.10	61	2	1422	0	1056	35.11	±.02	17.02
29	2	1307	212	1049	18.33	±.01	9.10	62	1	1428	0	1101	37.39	----	17.54
30	1	1308	187	1039	18.15	-----	9.16	63	1	1429	0	1101	37.38	----	17.63
31	3	1308	231	1041	18.15	±.01	9.16	64	5	1430	0	1100	37.68	±.19	17.72
32	1	1316	0	1049	20.36	-----	9.59	65	1	1431	0	1101	36.92	----	17.81
33	2	1317	0	1051	20.18	±.08	9.64	66	3	1449	0	1145	41.58	±.10	19.49

[a]Calculated from eq. (4).

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